Int J Theor Phys (2010) 49: 1706–1711
DOI 10.1007/s10773-010-0350-1

Black Holes in Accelerated Universe

Mubasher Jamil

Received: 29 January 2010 / Accepted: 6 April 2010 / Published online: 14 May 2010

Abstract We have analyzed the evolution of mass of a stationary black hole in the standard FRW cosmological model. The evolution is determined specifically about the time of transition from the earlier matter to the later exotic dark energy dominated universe. It turns out that the accretion rate of matter on the black hole of mass was approximately $O(10^{20})$ higher than the accretion rate of exotic dark energy at the time of transition.

Keywords Black hole · Dark energy · Phantom crossing

1 Introduction

The formation and evolution of supermassive black holes (SMBH) with masses in the range $10^6 - 10^9 M_\odot$, which mostly reside in the centers of galaxies is still an open problem [1–5]. It is generally accepted that these black holes had been formed by the accretion of both baryonic and dark matter onto the initial black hole seeds [6, 7]. One hypothesis is that these seed black holes were primordial black holes (of mass $\sim 100\ M_\odot$) formed from the death of the supermassive stars in the early universe [8]. The growth of these smaller black hole seeds would have been slow in the radiation dominated era while it would be significantly large in the matter dominated phase. In this later stage, the black hole seeds may had grown by several orders of magnitude in mass through the acquisition of large dark matter halos. Another model proposed by Hu et al. [9] divides the evolution history of black holes into two phases: the first phase starts with the quasi-spherical or Bondi accretion of mainly dark and baryonic matter onto the seed black hole at redshift $z \sim 30$ while the second phase deals with the accretion of pure baryonic matter which starts at $z \sim 6$ which goes on till now. Another scenario proposes the gravitational core collapse of relativistic star clusters which may have produced in the star-bursts at early times [10]. It is also suggested that SMBH

M. Jamil (✉)
Center for Advanced Mathematics and Physics, National University of Sciences and Technology, Rawalpindi, 46000, Pakistan
e-mail: mjamil@camp.nust.edu.pk

could have formed in the nuclei of proto-galaxies by the direct collapse of gas clouds in the pre-galactic halos, without invoking the primordial black hole seeds [11].

The standard model of cosmology based on the Friedmann-Robertson-Walker (FRW) spacetime predicts a dynamic history and the future of the observable universe. The observable universe evolves from the initial matter to the later exotic 'dark energy' dominated universe. Current astrophysical observations suggest that the universe is undergoing accelerated expansion [12–14] supporting the evidence of exotic vacuum energy commonly termed 'dark energy' having negative pressure $p < 0$ and positive energy density $\rho > 0$ persisting on the cosmological scale. The dark energy has been explained as cosmological constant [15], quintessence [16], holographic dark energy [17], tachyon [18], quintom model [19, 20], interacting dark energy model [21, 22] and modified gravity [23] to name a few. More exotic form of vacuum energy is the phantom energy [24] which possesses some weird properties: its accretion on the black hole can reduce its mass and appearance of naked singularities [25–27]; super-luminal and sometime negative sound speed [28]; decay of phantom field into matter [29]; increasing energy density with cosmic time and emergence of a future space-like singularity dubbed 'big rip' [30, 31]; existence of wormholes [32] and associated negative temperature with it [33]. Thus if the universe is pervaded with the phantom energy, the universe will have finite lifetime t^* (see [34, 35] for opposite viewpoint). Phantom energy can also be visualized as a scalar field ϕ with a non-canonical negative kinetic energy term $\dot{\phi}^2 < 0$ in the corresponding Lagrangian $L_\phi = (1/2)\partial^\mu\phi\partial_\mu\phi - V(\phi)$, where $V(\phi)$ is the potential term [36].

Thus from the above discussion it is safely drawn that the growth of black holes took place mostly in the matter dominated phase while in the phantom dominated phase, the black holes ultimately lose mass. We are particularly interested in the evolution of stationary and spherically symmetric black holes at the time of transition from the matter to the dark energy dominated universe. Although it can be anticipated that the effects of this transition will not be much significant and obvious at the astrophysical scales, this study at least gives us an estimate of the magnitude of the effects.

2 The model of accretion

We start by assuming the background to be spatially flat, homogeneous and isotropic FRW spacetime

$$ds^2 = -dt^2 + a^2(t)\left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]. \tag{1}$$

Here $a(t)$ is the dimensionless scale factor. The corresponding stress energy tensor is

$$T_{\mu\nu} = (\rho_m + \rho_x + p_x)u_\mu u_\nu + p_x g_{\mu\nu}, \tag{2}$$

where m and x refer to matter and the exotic vacuum energy respectively. The dynamical equations for two component fluid i.e. matter and the exotic vacuum energy are

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_x), \tag{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}[\rho_m + \rho_x(1+\omega)], \tag{4}$$

where we have used $p_x = \omega\rho_x$ for the phantom energy. The scale factor $a(t)$ evolves as [37]

$$a(t) = \frac{a(t_n)}{[-\omega + (1+\omega)\frac{t}{t_n}]^{-\frac{2}{3(1+\omega)}}} \quad \text{for } t > t_n, \tag{5}$$

where

$$t_n = \left(H_o\sqrt{\Omega_{m_o}}\right)^{-1}\left[\frac{2}{3}(1+z_n)^{-3/2}\right], \tag{6}$$

is the time for the transition from the earlier matter to the later dark energy dominated universe and z_n is the corresponding redshift of the transition. The Hubble's constant H_o determines the rate of expansion of the universe while Ω_{m_o} denotes the dimensionless density parameter of matter component. The transition time t_n is related with the current age of the universe t_o as

$$t_n = t_o\left[1 + \frac{(1+z_n)^{3(1+\omega)/2} - 1}{1+\omega}\right]^{-1}. \tag{7}$$

Notice that in the big rip scenario, the scale factor $a(t)$ diverges when the quantity in the square brackets in (5) vanishes identically i.e.

$$t^* = \frac{\omega}{1+\omega}t_n. \tag{8}$$

Subtracting t_n from (8), we get

$$t^* - t_n = -\frac{1}{1+\omega}t_n. \tag{9}$$

A black hole accreting only the exotic phantom energy has the following rate of change in mass [25, 38]

$$\left.\frac{dM}{dt}\right|_x = \frac{16\pi G^2}{c^5}M^2(\rho_x + p_x), \tag{10}$$

It is clear that when $\rho_x + p_x < 0$ which in turn implies $\omega < -1$, the mass of black hole will decrease. Also note that this same condition refers to violation of the so-called the null energy condition ($\rho + p > 0$). Since this condition is the weakest of all the energy conditions (weak, strong and dominant), it implies that the remaining conditions will also be violated. Moreover, the origin of this exotic phantom energy is still not clear but it is motivated from the astrophysical data which gives a convincing evidence of an evolving ω drifting towards more negative values [24, 39, 40]. The evolution of energy density of the exotic dark energy is given by

$$\rho_x^{-1} = 6\pi G(1+\omega)^2(t^* - t)^2. \tag{11}$$

We are particularly interested in the evolution of black hole about and after $t = t_n$ to analyze the affects of dark energy on the black hole. Using (9) and (11) in (10), we get

$$\left.\frac{dM}{dt}\right|_x = \frac{8G}{3c^3}\frac{M^2}{t_n^2}(1+\omega). \tag{12}$$

Therefore the mass change rate for a black hole accreting pure exotic energy is determined from (12). Note that the standard cosmological model based on the Friedman-Robertson-Walker spacetime is a hierarchal one: the universe starts from a radiation dominated phase

$\omega = 1/3$ and henceforth evolves successively to matter ($\omega = 0$), quintessence ($\omega > -1$), cosmological constant ($\omega = -1$) to a recent phantom energy dominated phase ($\omega < -1$) [39]. Thus in the FRW model, one cosmic specie dominates over all the others in a given phase i.e. the contribution from the all the other species is negligible. Also when the universe 'enters' in a new phase (i.e. shift of parameter ω to a new value), the previous value of ω no longer holds. Therefore when the universe enters the phantom regime, any previous values of ω is not relevant. Hence we cannot use $\omega = 0$ for matter in the phantom dominated regime. Observations also show that these shifts in ω are rather abrupt in the cosmic history. For instance, the matter-radiation decoupling occurred at a redshift $z \sim 1100$ almost all of a sudden and thermal equilibrium disappeared which yielded a matter dominated phase at later times. Similarly in recent times, the transition from an earlier deceleration phase ($\omega > -1$) to an acceleration phase ($\omega < -1$), the so-called phantom crossing [41–43], is also quite sudden in the perspective of cosmological time scales.

In order to study the evolution of a black hole in an epoch different from the matter dominated one, we cannot use $\omega = 0$ in (11) or (12) since these refer to only phantom energy. Now we consider the matter accretion onto the black hole. It is obtained by equating the Eddington and the accretion luminosities of a black hole to get

$$\left.\frac{dM}{dt}\right|_m = \frac{4\pi c m_p R}{\sigma_T}, \tag{13}$$

where $m_p \sim 1.67 \times 10^{-27}$ kg is the mass of the proton and $\sigma_T \sim 6.6 \times 10^{-28}$ m^2 is the Thomson cross-section. It should be mentioned that the Eddington accretion mechanism for matter is based on several assumptions which can explain accretion process to a good approximation. It requires inherently that the central accreting object is completely spherically symmetric; radiation output (luminosity) is isotropic; time independence (stationarity); negligible gas pressure etc. [44, 45]. However the spherical Eddington accretion model describes the accretion for black holes growth quite well. The Schwarzschild radius R of a stationary black hole is given by

$$R = \frac{2GM}{c^2}. \tag{14}$$

Using (14) in (13), we obtain

$$\left.\frac{dM}{dt}\right|_m = \frac{8\pi G}{c}\frac{Mm_p}{\sigma_T}. \tag{15}$$

Thus the evolution of black hole is determined by the contributions from (12) and (15) i.e.

$$\left.\frac{dM}{dt}\right|_{Tot} = \left.\frac{dM}{dt}\right|_m + \left.\frac{dM}{dt}\right|_x \tag{16}$$

$$= \frac{8\pi G}{c}\frac{Mm_p}{\sigma_T} + \frac{8G}{3c^3}\frac{M^2}{t_n^2}(1+\omega). \tag{17}$$

In the cosmological constant dominated universe $\omega = -1$, the second term in (17) will vanish while for a phantom energy dominated universe $\omega < -1$, the total accretion rate will be reduced. It can be seen that for a given value of $\omega < 0$ (say $\omega = -1.5$) and a black hole of mass M, (17) reduces to $1.4 \times 10^{-16} M - 4.9 \times 10^{-71} M^2$ for $t_n = 0.6 H_o^{-1}$, where

$H_o^{-1} = 13.65\ G\text{yr}$. A comparison of the two accretion rates implies

$$\eta(M) \equiv \frac{\dot{M}_m}{\dot{M}_x}. \tag{18}$$

For a black hole of mass 10^{31} kg, the dimensionless parameter η was approximately 10^{24} while for supermassive black holes of mass 10^{37} kg, it was roughly 10^{17}. Thus the contribution of dark energy was almost negligible compared to the accretion of matter during the time of transition showing that black holes remained largely unaffected at this epoch.

The evolution of mass of black hole after the transition period t_n to the big rip t^*, we can use (8) and (9) to get

$$\left.\frac{dM}{dt}\right|_x = \frac{8G}{3c^3}\frac{M^2}{(1+\omega)(t^*-t)^2}. \tag{19}$$

Again $1+\omega < 0$ leads to decreasing mass of BH where $t_n < t < t^*$. We are not interested in the case $t = t^*$, since it gives divergence in the rate of change in mass of BH and no physical conclusion can be drawn henceforth. The consequent evolution of BH is governed by the sum of (15) and (19) as

$$\left.\frac{dM}{dt}\right|_{Tot} = \left.\frac{dM}{dt}\right|_m + \left.\frac{dM}{dt}\right|_x \tag{20}$$

$$= \frac{8\pi G}{c}\frac{Mm_p}{\sigma_T} + \frac{8G}{3c^3}\frac{M^2}{(1+\omega)(t^*-t)^2}. \tag{21}$$

In this paper we have investigated the evolution of a black hole in the FRW universe. The mass of any black hole increases by the accretion of matter while it will decrease by the corresponding accretion of phantom energy. We have here studied the combined effects of this accretion at the moment of transition from the earlier matter dominated regime to the phantom regime. The rate of change in mass due to matter accretion is evaluated using the Eddington accretion model since cosmological matter density decreases faster. It turns out that the effects of the accretion of matter was large while the phantom accretion was negligibly small. Hence the effects of transition were negligible at the instant of transition.

References

1. Ferrarese, L., Merritt, D.: Astrophys. J. Lett. **539**, 9 (2000)
2. Volonteri, M.: arXiv:astro-ph/0602630v1
3. J.-M., Wang, et al.: arXiv:0712.0523v1 [astro-ph]
4. Begelman, M.C.: Science **300**, 1898 (2003)
5. Miller, M.C.: Mon. Not. R. Astron. Soc. Lett **367**, L32 (2006)
6. Duching, N.: Phys. Rev. D **70**, 064015 (2004)
7. Ostriker, J.P.: Phys. Rev. Lett. **84**, 5258 (2000)
8. Mack, K.J.: arXiv:astro-ph/0608642v1
9. Hu, J., et al.: Mon. Not. R. Astron. Soc. **365**, 345 (2006)
10. Shapiro, S.L.: Astrophys. J. **613**, 1213 (2004)
11. Begelman, M.C., et al.: Mon. Not. R. Astron. Soc. **370**, 289 (2006)
12. Perlmutter, S., et al.: Astrophys. J. **517**, 565 (1999)
13. Riess, A.G., et al.: Astron. J. **116**, 1009 (1998)
14. Spergel, D.N., et al.: Astron. J. Suppl. **170**, 377 (2007)
15. Padmanabhan, T.: Phys. Rep. **380**, 235 (2003)
16. Caldwell, R.R., et al.: Phys. Rev. Lett. **80**, 1582 (1998)

17. Setare, M.R.: Eur. Phys. J. C **50**, 991 (2007)
18. Padmanabhan, T.: Phys. Rev. D **66**, 021301 (2002)
19. Cai, Y.-F., et al.: Phys. Lett. B **646**, 141 (2007)
20. Cai, Y.-F., et al.: Phys. Lett. B **651**, 1 (2007)
21. Jamil, M., Rashid, M.A.: Eur. Phys. J. C **58**, 111 (2008)
22. Jamil, M., Rashid, M.A.: Eur. Phys. J. C **56**, 429 (2008)
23. Brevik, I.: Gen. Relativ. Gravit. **38**, 1317 (2006)
24. Caldwell, R.R.: Phys. Lett. B **545**, 23 (2002)
25. Babichev, E., et al.: Phys. Rev. Lett. **93**, 021102 (2004)
26. Jamil, M., et al.: Eur. Phys. J. C **58**, 325 (2008)
27. Nesseris, S., Perivolaropoulos, L.: Phys. Rev. D **70**, 123529 (2004)
28. Gonzalez-Diaz, P.F.: arXiv:astro-ph/0312579v1
29. Baushev, A.: Phys. Lett. B **684**, 69 (2010)
30. Caldwell, R.R., et al.: Phys. Rev. Lett. **91**, 071301 (2003)
31. Dabrowski, M.P.: arXiv:gr-qc/0701057v1
32. Jamil, M.: Il Nuovo Cimento B (2008). doi:10.1393/ncb/i2008-10555-1
33. Gonzalez-Diaz, P.F., Siguenza, C.L.: Nucl. Phys. B **697**, 363 (2004)
34. Gonzalez-Diaz, P.F.: Phys. Rev. D **68**, 021303 (2003)
35. Srivastava, S.K.: arXiv:0707.1376v2 [gr-qc]
36. Urena-Lopez, L.A.: J. Cosmol. Astropart. Phys. **09**, 013 (2005)
37. Johri, V.: Phys. Rev. D **70**, 041303(R) (2004)
38. Jamil, M., Qadir, A.: Gen. Relativ. Gravit. To appear. arXiv:0908.0444v3 [gr-qc]
39. Carroll, S.M., et al.: Phys. Rev. D **68**, 023509 (2003)
40. Feng, B., et al.: Phys. Lett. B **607**, 35 (2005)
41. Wu, S.-F., et al.: Phys. Lett. B **659**, 45 (2008)
42. Wei, H., Cai, R.-G.: Phys. Lett. B **634**, 9 (2006)
43. Amendola, L., Tsujikawa, S.: Phys. Lett. B **660**, 125 (2008)
44. Heinzeller, D., Duschl, W.J.: Mon. Not. R. Astron. Soc. **374**, 1154 (2007)
45. Heinzeller, D., et al.: arXiv:astro-ph/0610544v1